

# On duality of Drell-Yan and $J/\psi$ production processes

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## Abstract

It is studied the model on  $J/\psi$  production allowing to extract parton distribution functions (PDFs) from the combined analysis with both data on Drell-Yan and  $J/\psi$  production processes. It is shown that this, so attractive from theoretical point of view, model, can be safely used in the low energy region  $E \lesssim 100 GeV$ . The significance of gluon contributions to the  $J/\psi$  cross-sections is investigated. The obtained results in the high energy region occur to be rather surprising.

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Nowadays we see the growing interest [1, 2] to the close analogy (duality) between Drell-Yan (DY)  $H_1 H_2 \rightarrow \gamma^* X \rightarrow l^+ l^- X$  and  $J/\psi$   $H_1 H_2 \rightarrow J/\psi X \rightarrow l^+ l^- X$  production mechanisms (see textbook [3] for review). It is assumed that a such analogy/duality occurs at relatively low energies, when the gluon-gluon fusion ( $gg$ ) mechanism of  $J/\psi$  production is dominated by the quark-antiquark fusion ( $\bar{q}q$ ). Then, since  $J/\psi$  is a vector particle like  $\gamma$  and the helicity structure of  $\bar{q}q(J/\psi)$  and  $(\bar{q}q)\gamma^*$  couplings is the same, one can get the  $J/\psi$  production cross-section from the DY process cross-section applying the simple replacement

$$16\pi^2 \alpha^2 e_q^2 \rightarrow (g_q^{J/\psi})^2 (g_\ell^{J/\psi})^2, \quad \frac{1}{M^4} \rightarrow \frac{1}{(M^2 - M_{J/\psi}^2)^2 + M_{J/\psi}^2 \Gamma_{J/\psi}^2}, \quad (1)$$

where  $M^2 \equiv Q^2$  is the squared mass of dilepton pair,  $M_{J/\psi}^2 \simeq 9.59 GeV^2$  is the squared  $J/\psi$  mass and  $\Gamma_{J/\psi}$  is the full  $J/\psi$  width. It is believed that the model (1) can be applied in both unpolarized [2] and polarized [1] cases. The later is due to the identical helicity and vector structure of  $\gamma^*$  and  $J/\psi$  elementary channels (all  $\gamma^\mu$  couplings). In particular, the model given by Eq. (1) was exploited in Ref. [1] in the double-polarized (both hadrons in initial state are transversely polarized) case.

It should be noticed here, that though the pure vector  $\gamma_\mu$  coupling (Dirac coupling) is very likely in the  $J/\psi$  production, still it is a hypothesis. In principle, still the effects due to  $\sigma_{\mu\nu}$  coupling (Pauli coupling) are possible. Later, for the number of processes with the different *unpolarized* colliding hadrons it will be shown that the model (1), based on pure vector coupling, indeed works well at low energies. It can be considered as a strong argument in favor of pure vector coupling mechanism. At the same time, in *polarized* case we still have no necessary data to check the pure vector coupling hypothesis. Thus, for the comprehensive verification of the “duality” model (1) it is of crucial importance to collect the data on  $J/\psi$  production with the polarized colliding hadrons. This is a strong additional motivation to perform the such experiments. At present the  $J/\psi$  production processes with polarized proton/deuteron target are planned at J-PARC [4] (proton beam) and COMPASS [5] (pion beam). The respective experiment with the polarized antiproton beam is possible at GSI (this possibility is now under discussion [6]).

The advantage of model (1) is that in the region of  $u$ -quark dominance (large Bjorken  $x$ ) all couplings exactly cancel out in the ratios of cross-sections (like asymmetries), so that they

become absolutely the same for the DY and  $J/\psi$  production processes (see Eq. (23) in Ref. [1] and Eq. (10) in Ref. [2]). Thus, in this kinematical region it does not matter for the  $u$  quark PDFs extraction where do the dilepton pair production events come from: from continuum or from  $J/\psi$  production region. Certainly, the such possibility to use  $J/\psi$  production for PDFs extraction is very attractive because the dilepton production rate in the  $J/\psi$  production region is two orders of magnitude higher than in the continuum region above the  $J/\psi$  mass. In particular, the model (1) give us the possibility to extract transversity  $h_{1u}$  and the first moment of Boer-Mulders function  $h_{1u}^{\perp(1)}$  from  $J/\psi$  production region. Namely, in the large  $x$  region the equations (Eqs. (19), (20) in Ref. [7] and Eqs. (12), (13) in Ref. [8]) for unpolarized  $H_1 H_2 \rightarrow \gamma^* X \rightarrow l^+ l^- X$

$$\hat{k} = 8 \frac{h_{1\bar{u}}^{\perp(1)}(x_A)|_A h_{1u}^{\perp(1)}(x_B)|_B + (u \leftrightarrow \bar{u})}{f_{1\bar{u}}(x_A)|_A f_{1u}(x_B)|_B + (u \leftrightarrow \bar{u})} \quad (2)$$

and single-polarized  $H_1 H_2^\uparrow \rightarrow \gamma^* X \rightarrow l^+ l^- X$

$$\hat{A} = -\frac{1}{2} \frac{h_{1\bar{u}}^{\perp(1)}(x_A)|_A h_{1u}(x_{B^\uparrow})|_{B^\uparrow} + (u \leftrightarrow \bar{u})}{f_{1\bar{u}}(x_A)|_A f_{1u}(x_{B^\uparrow})|_{B^\uparrow} + (u \leftrightarrow \bar{u})} \quad (3)$$

DY processes are the same for the respective  $J/\psi$  production processes  $H_1 H_2 \rightarrow J/\psi X \rightarrow l^+ l^- X$  and  $H_1 H_2^\uparrow \rightarrow J/\psi X \rightarrow l^+ l^- X$ , providing us an access to  $h_{1u}$  and  $h_{1u}^{\perp(1)}$  as a result of combined analysis of DY and  $J/\psi$  data.

Certainly, the “duality” model (1) is applicable only in the such kinematical regions where among the elementary processes contributing to  $J/\psi$  production the quark-antiquark fusion process dominates over the gluon-gluon fusion. Qualitatively, it is clear that the gluon contributions should be suppressed at low energies: at fixed  $Q^2$  value ( $9.6 \text{ GeV}^2$  for  $J/\psi$  production we deal with)  $x$  increase when  $s \equiv (p_{H_1} + p_{H_2})^2$  decreases, while the ratio  $g(x)/q(x)$  sharply decreases when  $x$  increases. However, we certainly need the quantitative analysis to find the region where we can safely neglect the gluons. The main goal of this paper is to perform the such quantitative analysis. To this end we, besides of the model (1), will consider the most popular and well established model on  $J/\psi$  production which includes all elementary processes (quark-antiquark as well as gluon-gluon fusion) and believed to be valid at any energies of incident hadrons – see [9] and references therein. Then we will compare the predictions of both these models with the respective existing experimental data [10]-[16].

To cancel unknown constants, we will study not absolute cross-sections but the ratios<sup>1</sup> of cross-sections corresponding to the variety of hadrons/nucleus in initial state. Namely, we will consider the ratios of the angle and  $x_F$  integrated (the later in the forward hemisphere  $x_F > 0$ ) cross-sections on  $J/\psi$  production  $\sigma_{pp}/\sigma_{\pi^\pm p}$ ,  $\sigma_{pA}/\sigma_{\pi^\pm A}$  and  $\sigma_{pp}/\sigma_{\bar{p}p}$ , where the symbol  $A$  denotes the different target nucleus (these are  $W$ ,  $C$ ,  $Ca$ ,  $Cu$ ,  $Pt$  here). The point is that the hadrons  $\pi^\pm$  and  $\bar{p}$  (on the contrary to proton and nuclei) contain the antiquark in the valence state. That is why it is very important to study the ratios like  $\sigma_{pp}/\sigma_{\pi^\pm p}$  and  $\sigma_{pp}/\sigma_{\bar{p}p}$  which should possess very specific behavior: they should sharply decrease when  $s$  decrease (large  $x$ , valence quark/antiquark dominance region) and they should increase up to unity when  $s$  takes large values (small  $x$ , sea quark and gluon dominance region). The most of data on  $J/\psi$  production giving us an access to the such kind of ratios are the data in the forward hemisphere with the pion or antiproton beams colliding with the proton or nuclear targets [10]-[16].

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<sup>1</sup>Let us recall that as a final goal we are interested namely in the ratios – asymmetries, which can give us an access to the different PDFs.

The simple “duality” model (1) applied to the ratio of cross-sections  $[\sigma_{H_1 H_2}/\sigma_{H'_1 H_2}]_{x_F>0}$  gives

$$[\sigma_{H_1 H_2}/\sigma_{H'_1 H_2}]_{x_F>0} = \frac{\int_0^{1-m_{J/\psi}^2/s} dx_F [s(x_1 + x_2)]^{-1} F_{q\bar{q}}^{H_1 H_2}}{\int_0^{1-m_{J/\psi}^2/s} dx_F [s(x_1 + x_2)]^{-1} F_{q\bar{q}}^{H'_1 H_2}}, \quad (4)$$

where the quark/antiquark flow  $F_{q\bar{q}}^{H_1 H_2}$  is given by

$$F_{q\bar{q}}^{H_1 H_2} = \sum_{q=u,d,s} [q^{H_1}(x_1)\bar{q}^{H_2}(x_2) + \bar{q}^{H_1}(x_1)q^{H_2}(x_2)], \quad (5)$$

and  $x_{1,2}$  are expressed via  $x_F = x_1 - x_2$  as  $x_{1,2} = [\pm x_F + \sqrt{x_F^2 + 4m_{J/\psi}^2/s}]/2$ . Notice that obtaining Eqs. (4), (5) we put the ratios  $g_d^{J/\psi}/g_u^{J/\psi}$  and  $g_s^{J/\psi}/g_u^{J/\psi}$  equal to unity in accordance with the existing experimental data [17]. Indeed, the data [17] on  $J/\psi$  production with the absolutely symmetric (so that the cross section per nucleon reads  $\sigma_{\pi^\pm C} = [\sigma_{\pi^\pm p} + \sigma_{\pi^\pm n}]/2$ ) carbon target clearly indicates that the ratio  $\sigma_{\pi^+ C}/\sigma_{\pi^- C}$  is seem to be near unity in  $J/\psi$  region, while it falls toward 1/4 above the dilepton pair mass – see Fig. 2 in Ref. [17] and discussion around. This is a good argument that on the contrary to  $q\bar{q}$  annihilation mechanism for DY process (where  $d$  quark is suppressed by the factor  $e_d^2/e_u^2 = 1/4$ ), for  $J/\psi$  production  $u$  and  $d$  quarks should enter the cross-sections with the same charge factors:  $g_d^{J/\psi}/g_u^{J/\psi} \simeq 1$ . We also use the analogous relation  $g_s^{J/\psi}/g_u^{J/\psi} \simeq 1$  keeping in mind that the squared sea strange quark contribution is not so significant.

The main difference of the “gluon evaporation” model from the model (1) is that the former in addition to  $q\bar{q}$  fusion contribution  $F_{q\bar{q}}^{H_1 H_2}$  given by Eq. (5) contains also gluon-gluon fusion contribution  $F_{gg}^{H_1 H_2} = g^{H_1}(x_1)g^{H_2}(x_2)$ . Applied to the ratios  $[\sigma_{H_1 H_2}/\sigma_{H'_1 H_2}]_{x_F>0}$ , the “gluon evaporation” model produces:

$$\frac{\sigma_{H_1 H_2}|_{x_F>0}}{\sigma_{H'_1 H_2}|_{x_F>0}} = \frac{(\sigma_{q\bar{q}} + \sigma_{gg})_{H_1 H_2}|_{x_F>0}}{(\sigma_{q\bar{q}} + \sigma_{gg})_{H'_1 H_2}|_{x_F>0}}, \quad (6)$$

$$\sigma_{q\bar{q}(gg)}^{H_1 H_2}|_{x_F>0} = \int_{4m_c^2}^{4m_D^2} dQ^2 \int_0^{1-\frac{Q^2}{s}} dx_F \sigma^{q\bar{q} \rightarrow c\bar{c}(gg \rightarrow c\bar{c})}(Q^2) \frac{x_1 x_2}{Q^2(x_1 + x_2)} F_{q\bar{q}(gg)}^{H_1 H_2}, \quad (7)$$

where  $2m_c = 3.0 \text{ GeV}$  and  $2m_D = 3.74 \text{ GeV}$  are respectively the  $c\bar{c}$  and open charm thresholds, while the elementary cross-sections  $\sigma^{q\bar{q} \rightarrow c\bar{c}}$ ,  $\sigma^{gg \rightarrow c\bar{c}}$  are proportional to  $\alpha_s(Q^2)$  and can be found, for example, in Ref. [9] (see Eqs. (3) and (4)). For the comparison purposes we will consider also the “gluon evaporation” model without gluon contribution  $F_{gg}^{H_1 H_2}$ . It is obvious that it differs from the “duality” model only by the extra integration over  $Q^2$  with the weight  $\sigma^{q\bar{q} \rightarrow c\bar{c}}(Q^2)$ .

Let us first consider the ratios  $\sigma_{pp}/\sigma_{\pi^\pm p}$ . The results obtained within the “duality” and “gluon evaporation” models are presented in Fig.1 in comparison with experimental data. First of all, one can conclude that in the low energy region, near the first experimental point  $\sqrt{s} \simeq 8.7 \text{ GeV}$ , the curves corresponding to “duality” model and to “gluon evaporation” model with and without gluons almost merge and equally well describe the existing experimental data. This is not suprising and is in agreement with the qualitative predictions: as it was discussed above, the gluon contribution should be suppressed in the low energy region. At the same time, the results in the high energy region occur to be rather surprising: even at very high energies (150 GeV and 200 GeV) the gluon contribution seems to be insignificant in the ratios  $\sigma_{pp}/\sigma_{\pi^\pm p}$  and the curves with and without gluon contributions  $F_{gg}^{H_1 H_2}$  equally well describe the existing

data. Thus, in the case of  $150\div 200$  GeV pion beam and the proton target we need to improve the quality of data on  $J/\psi$  production to distinguish between the models with and without gluon contribution. The such measurements can be performed, for example, by COMPASS collaboration [5], where the possibility to study Drell-Yan and  $J/\psi$  processes with the pion beam is now under consideration.

The absolutely analogous picture, insignificance of the gluon contribution even at high energies, occur also for the ratios  $\sigma_{pA}/\sigma_{\pi^\pm A}$  with the different target nuclei – see Fig.2. In Fig.2 the data with the approximately equal ratio  $Z/A \simeq 0.4$  are collected. The curves corresponding to the model calculations are obtained with  $Z/A = 0.4$  and neglecting<sup>2</sup> the nuclear effects, so that the cross-section per nucleon reads:  $\sigma_{hA}|_{h=\pi^\pm, p} = \frac{Z}{A}\sigma_{hp} + \left(1 - \frac{Z}{A}\right)\sigma_{hn}$ .

Let us now consider the ratios  $\sigma_{pp}/\sigma_{\bar{p}p}$  with the incident antiproton instead of pion. The results are presented in Fig.3. While in the low energy region we again see the good agreement between the models with and without gluons and the data (as it should be from the qualitative consideration), the situation in high energy region is absolutely different. First, one can conclude, that the gluon contribution becomes very significant in this kinematical region. Second, and rather surprising conclusion, is that the widely used “gluon evaporation” model works rather bad in this case – the respective curve lies well below the data (bold solid line in Fig. 3). Notice that this result is in a strong disagreement with the statement made in Ref. [15], where the same experimental points were used. Indeed, calculations within the “gluon evaporation” model presented in Ref. [15] (bold dashed line in Fig.3) are in a good agreement with the data points which was claimed there as a strong argument in favor of that model (see Fig.6 in Ref. [15] and discussion around). The reason of this disagreement is in the gluon<sup>3</sup> sector of the model, because the main difference of the parametrization [18] used in Ref. [15] and the modern parametrizations<sup>4</sup> used in our calculations is just the gluon distribution values. Certainly, one should trust namely the calculation where the modern parametrizations are used, because  $G(x)$  is much better fixed there by a lot of new data appeared since the paper [18] issue. The respective results occur to be in the strong disagreement with the data and, thus, one can conclude that to pass this test the gluon sector in the “gluon evaporation” model should be essentially modified. Besides, all existing nowadays models on  $J/\psi$  production should also pass this test on high energy behavior, and this is a subject of our future investigation.

In conclusion, we have tested the “duality” model as well as “gluon evaporation” model in the different energy ranges. It is shown that the “duality” model (as well as “gluon evaporation” model) works well in the low energy sector,  $s \lesssim 100\text{GeV}^2$ . In this region the curves with and without gluon contributions almost coincide and equally well describe the data. Thus, we can safely apply in this kinematical region so attractive from the theoretical point of view “duality” model. This give us the unique possibility to use the  $J/\psi$  production region for combined analysis with the data on Drell-Yan processes to extract PDFs we are interested in. This is an excellent possibility to essentially increase the statistics since the  $J/\psi$  production cross-sections are in about two orders of magnitude higher than the respective DY cross-sections.

On the other hand, we got two surprises in the high energy region. First is for the pion-

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<sup>2</sup>Usually [9] the nuclear effects are accumulated in the multiplier  $A^\alpha$ . However, for  $x_F$  integrated cross-sections these factors differ a little from unity – see Ref. [9] and references therein.

<sup>3</sup>It is clear seen from Fig. 3. Indeed, the curves corresponding to the “gluon evaporation” model without gluons, for both old and modern parametrizations (thin dashed and solid lines in Fig. 3), practically merge.

<sup>4</sup>We present here the result (bold solid line in Fig. 3) with the modern and widely used GRV98 [19] parametrization. However, our calculations with another modern parametrizations produce the same picture (the results differ a little from the respective results with the GRV98 parametrization).

proton/nuclei collisions, where the gluon contribution seems to be insignificant even at the energies of incident pion about  $150\div 200$  GeV. The second surprise occurs for the antiproton-proton collisions. Here, on the one hand, the gluon contribution in the ratio  $\sigma_{pp}/\sigma_{\bar{p}p}$  is very essential. On the other hand, it is not properly described by the popular and widely used “gluon evaporation” model, when we use the modern parametrizations on  $G(x)$  instead of old (and rather incorrect) one. Thus, it seems that the “gluon evaporation” model should be properly modified.

Thus, the results of performed tests definitely show that both theoretical and experimental efforts are still necessary to answer the raised questions. We need the new information from both low and high energy regions. In this respect it should be noticed that at present the new experiments with the proton-proton collisions on Drell-Yan and  $J/\psi$  processes are planned [4] at J-PARC facility. The respective measurements with the low energy antiproton beam could be performed at GSI (see Ref. [6]). At the same time, the researches on Drell-Yan and  $J/\psi$  physics at high energies ( $150\div 200$  GeV energy of incident pion) are now planned at COMPASS experiment [5].

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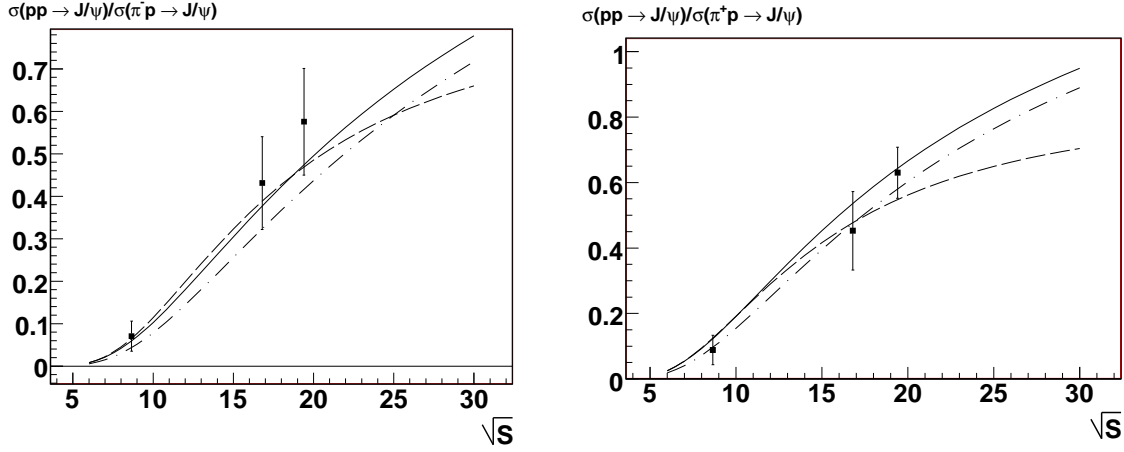


Figure 1: Ratios of cross-sections  $\sigma_{pp}/\sigma_{\pi^\mp p}$  on  $J/\psi$  production calculated with two models in comparison with the experimental data. Solid line corresponds to the “duality” model, Eqs. (4), (5). Dashed line corresponds to the “gluon evaporation” model, Eqs. (6), (7). Dot-dashed line corresponds to “gluon evaporation” model without gluon contribution. Here GRV94 [21] parametrization for the proton PDFs and GRV [20] parametrization for the pion PDFs are used. The experimental data (points with error bars) are taken from [9] (Tables 2 and 3).

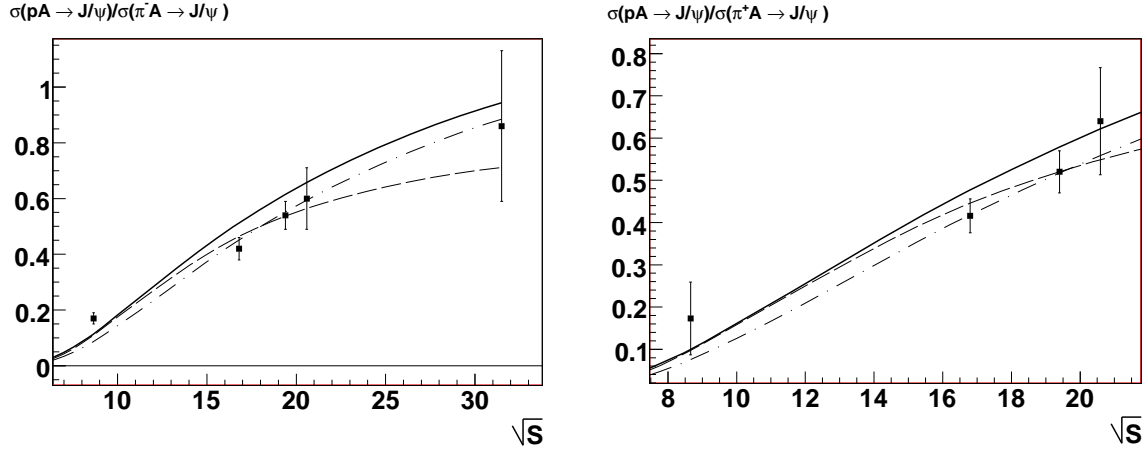


Figure 2: Ratios of cross-sections  $\sigma_{pA}/\sigma_{\pi^\mp A}$  on  $J/\psi$  production for the nuclear targets ( $Z/A \simeq 0.4$ ) calculated with two models in comparison with the experimental data. Solid line corresponds to the “duality” model, Eqs. (4), (5). Dashed line corresponds to the “gluon evaporation model”, Eqs. (6), (7). Dot-dashed line corresponds to the “gluon evaporation” model without gluon contributions. Here GRV94 [21] parametrization for proton PDFs and GRV [20] parametrization for pion PDFs are used. The points with error bars correspond to experimental data. First point: W,  $Z/A=0.40$  [11]; second and third points: Pt,  $Z/A=0.40$  [13]; fourth point: C,  $Z/A=0.5$  [16]; fifth point: Be,  $Z/A=0.44$  [14].

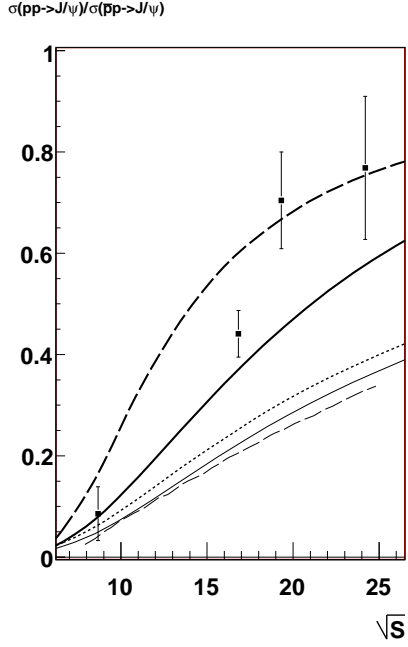


Figure 3: Ratios of cross-sections  $\sigma_{pp}/\sigma_{\bar{p}p}$  on  $J/\psi$  production for the proton target calculated with two models in comparison with experimental data. The experimental data are taken from Ref. [15], where the respective data of NA3, WA39 and UA6 collaborations were collected (see Fig.6 in Ref. [15]). The bold solid and dashed lines correspond to calculation with the “gluon evaporation” model (with GRV98 [19] and Duke-Owens [18] parametrizations, respectively). The thin solid and dashed lines correspond to calculations with the “gluon evaporation” model without gluon contributions (with GRV98 [19] and Duke-Owens [18] parametrizations, respectively). The dotted line corresponds to calculation with the “duality” model and GRV98 [19] parametrization.